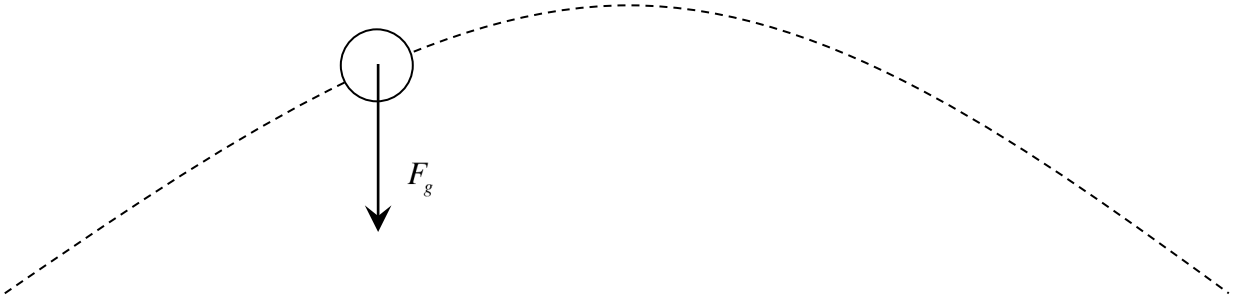
Projectile Motion

Introduction

When an object is thrown through the air it follows a curving path called a **trajectory**. A good description of the motion of a projectile can be obtained by ignoring the effects of air resistance.

Consider the path of a baseball as it moves through the air.



Draw a free body diagram of the forces acting on the ball at **any** point along its trajectory.

Notice the following:

$\sum \overrightarrow{F_x} = 0$ (no horizontal forces)

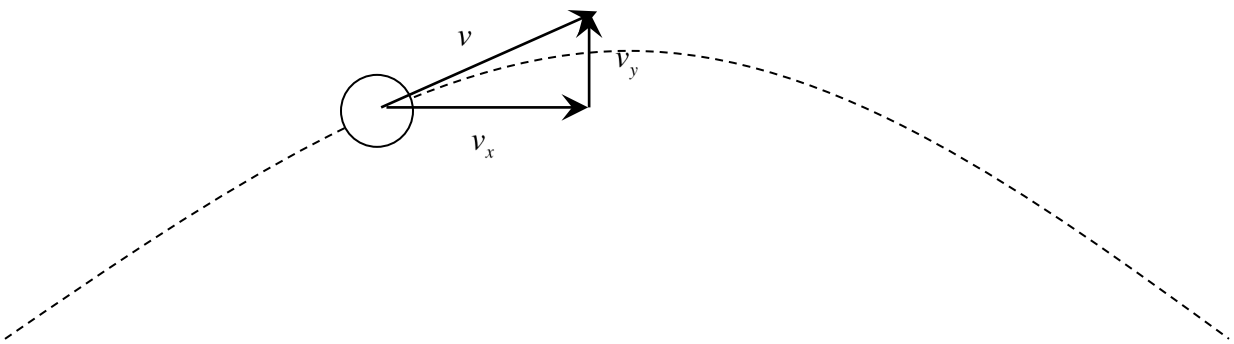
$\therefore \overrightarrow{a_x} = 0$ (**constant horizontal velocity**)

$\sum \overrightarrow{F_y} = \overrightarrow{F_g}$

$m\overrightarrow{a_y} = m\vec{g}$

$\therefore \overrightarrow{a_y} = \vec{g}$ (**vertical acceleration of** $9.8\ m/s^2\ [down]$)

Next, draw a vector diagram showing the velocity of the ball and its components. (**Note**: the velocity of the ball is tangent to its path)



Based on the conclusions from the free body diagram, we have the following rules for projectile motion:

1. The horizontal component of the projectile's velocity $\left(v_x\right)$ is constant.

2. In the vertical direction, the projectile undergoes constant acceleration $\left(a = -9.8\ m/s^2\right)$.

Thus, to solve projectile motion problems we must consider the horizontal and vertical directions separately.

Projectiles Launched Horizontally

The simplest form of projectile motion occurs when the projectile is launched horizontally. The diagram below illustrates this situation:

Based on our previous observations about projectiles, we can analyze the motion of the projectile. To do so, we must consider the horizontal and vertical components of the projectiles motion separately.

Horizontal **Vertical**

Example 1
A rock is thrown horizontally at $10.0\ m/s$ from the top of a cliff $122.5\ m$ high.

a. How long does it take the rock to reach the ground?

b. How far from the base of the cliff does the rock land?

Example 2

A ball is thrown from the top of a building with a horizontal velocity of $20.0\ m/s$. It hits level ground $80\ m$ from the base of the building. How high is the building?

Projectiles Worksheet #2

1. A projectile is launched horizontally from the top of a cliff $490\ m$ high with a speed of $100\ m/s$. Find its time of flight and its horizontal range. ($10\ s$, $1000\ m$)

2. A rifle which shoots a bullet at $1000\ m/s$ is fired horizontally at the center of a target $500\ m$ away. How far below the center does the bullet strike the target? ($1.23\ m$)

3. A rock is thrown horizontally at $15\ m/s$ from the top of a cliff $200\ m$ high.

   a) How long does the rock take to reach the ground? ($6.4\ s$)

   b) What is the range of the rock from the base of the cliff? ($95.8\ m$)

4. A stone is thrown horizontally from the top of a tall building takes $10\ s$ to reach the street below. How high is the building? ($490\ m$)

5. A bomber in level flight, flying at $100\ m/s$, releases a bomb at a height of $2000\ m$.

   a) How long is it before the bomb strikes the Earth? ($20.2\ s$)

   b) How far (horizontally) before the target must the bomber release the bomb? ($2020\ m$)

6. A cannon is arranged to fire a projectile with a horizontal speed of $200\ m/s$ from the top of a cliff overlooking a bay. The range of the cannon is $1200\ m$. How high is the cliff? With what velocity does the projectile strike the water? ($176.4\ m$; $208.5\ m/s\ \left[16°\ BTH\right]$)

7. A ball is thrown horizontally from a point near the edge of the roof of a building $200\ m$ tall, with a speed of $15\ m/s$. It reaches the ground in free flight.

   a) Determine the range of the ball from the base of the building. ($95.8\ m$)

   b) What velocity does the ball strike the ground at? ($64.4\ m/s\ \left[77°\ BTH\right]$)

   c) Determine the position of the ball at $t = 4\ s$. ($98.7\ m\ \left[53°\ BTH\right]$)

8. A ball is thrown horizontally from a point near the edge of the roof of a building $122.5\ m$ tall, with a speed of $20\ m/s$. It reaches the ground in free flight.

   a) How long does it take to reach the ground? ($5\ s$)

   b) How far from the vertical side of the building does the ball land? ($100\ m$)

   c) With what velocity does it strike the ground? ($52.9\ m/s\ \left[68°\ BTH\right]$)

   d) Determine the position of the ball at $t = 3\ s$. ($74.5\ m\ \left[36°\ BTH\right]$)

   e) Determine the velocity of the ball at $t = 3\ s$. ($35.6\ m/s\ \left[56°\ BTH\right]$)